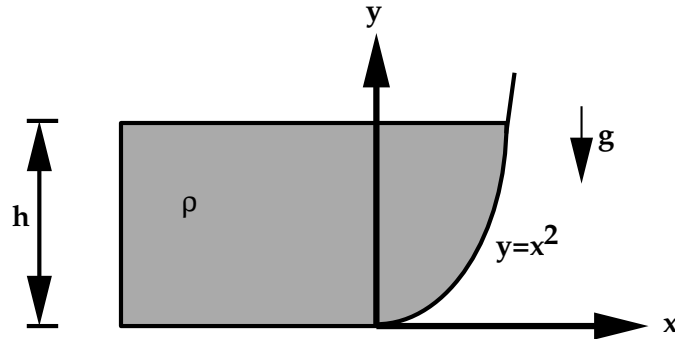
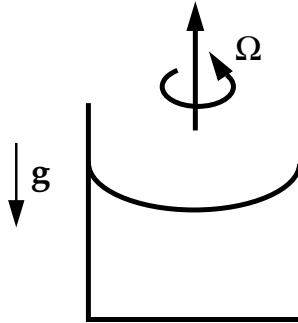


HYDROSTATICS

1.1) Determine the total force exerted by a mass of fluid at rest on the surface of a dam of parabolic shape, as indicated in the figure. The density of the fluid is ρ and the height of the fluid free surface from the bottom is h .



1.2) Determine the shape of the free surface of an incompressible fluid subject to a uniform gravitational field, contained in a cylindrical vessel which rotates about its (vertical) axis with constant angular velocity Ω .



1.3) Obtain an expression for the pressure at the center of a self-gravitating spherical star of which the density at a distance r from its center is (linear stellar model):

$$\rho = \rho_0(1 - \beta r^2)$$

Show that if the mean density be twice the surface density, the pressure at the center is greater by a factor $13/8$ than if the star had uniform density with the same total mass.

KINEMATICS

1.4) Find the stream function ψ of the vortex in cylindrical coordinates, given that its velocity field is:

$$v_r = v_z = 0, \quad v_\theta = \omega r$$

where ω is the angular velocity and assuming that $\psi = 0$ at $r = 0$. How do the streamlines look like ?

1.5) Find the velocity potential and the stream function in polar coordinates of:

a) a point source/sink

b) a point vortex

Make a sketch of the streamlines and the iso-potential lines.