DYNAMICS

2.1) For a Newtonian fluid with uniform viscocity under the action of conservative forces, derive the following equation for the vorticity:

$$\frac{D\underline{\omega}}{Dt} = \nu \nabla^2 \underline{\omega} + \underline{\omega} \cdot \nabla \underline{u} - \underline{\omega}(\nabla \cdot \underline{u})$$

2.2) Starting with the momentum conservation law, derive the integral equation for the conservation of angular momentum ($\underline{\Sigma} = \underline{\underline{\sigma}} \cdot \underline{n}$):

$$\frac{d}{dt} \int \underline{r} \times \rho \underline{u} dV = \oint \underline{r} \times \underline{\Sigma} dA + \int \underline{r} \times \underline{F} dV$$

2.3) Show that a conservation law written as

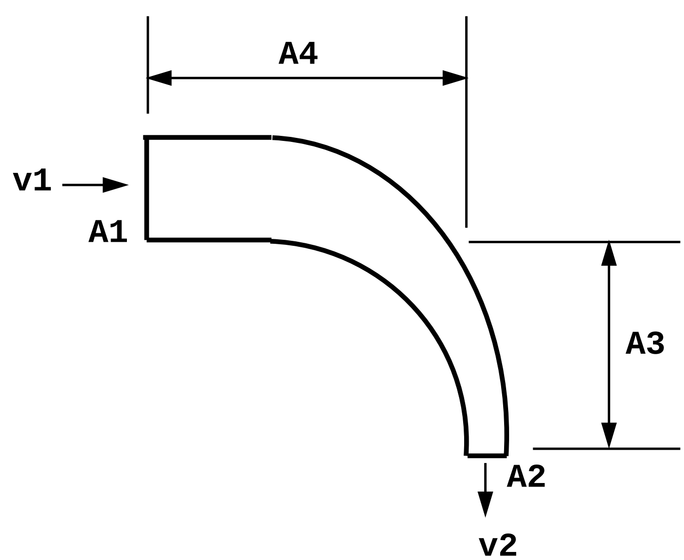
$$\frac{\partial}{\partial t}(\rho\theta) + \nabla \cdot (\rho\theta\underline{u}) = 0$$

is equivalent to

$$\rho \frac{D\theta}{Dt} = 0$$

2.4) An incompressible fluid is flowing steadily through a 90^0 reducing elbow. At the inlet the pressure is p_1 and the area A_1. At the outlet the area is A_2, the velocity v_2 and the pressure p_0 (ambient pressure). Determine the force necessary to hold the elbow in place, assuming uniform flow over each section.

Hint: use momentum theorem, and assume the area of the horizontal portion to be A_4 and the area of the vertical portion A_3, as indicated in the figure.



2.5) Consider a 2D jet of width b impinging on a wall at an angle θ, and dividing into two streams of widths b_1 and b_2, respectively. The ambient pressure is p_0. Find the force that must be applied to hold the wall in equilibrium.