TURBULENCE

5.1) Show that the incompressible continuity equation is satisfied by the instantaneous velocity fluctuations alone.

5.2) Defining the *Favre* average as:

$$\tilde{f} = \frac{\overline{\rho f}}{\overline{\rho}}$$

where the conventional average is

$$\overline{f} = \frac{1}{T}\int_0^T f\,dt$$

a function can be decomposed as a mean and fluctuating part:

$$f = \tilde{f} + f'$$

Besides the usual Reynolds conditions, the Favre average yields the following:

$$\overline{\tilde{f}} = \tilde{f}$$

$$\overline{\rho f'} = 0$$

$$\overline{f'} = -\overline{\rho' f'}/\overline{\rho}$$

$$\overline{\rho f g} = \overline{\rho}\tilde{f}\tilde{g} + \overline{\rho f' g'}$$

Taking the Favre average of the mass and momentum equations, derive the corresponding averaged equations:

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial \overline{\rho}\tilde{u}_i}{\partial x_i}$$

$$\frac{\partial \overline{\rho}\tilde{u}_i}{\partial t} + \frac{\partial \overline{\rho}\tilde{u}_i\tilde{u}_j}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(-\overline{\rho u_i' u_j'} + \overline{\tau}_{ij}) + \overline{F}_i$$