CSI 771 Problem Set 5

5.1 If

$$\begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{pmatrix} 5 \\ 12 \end{pmatrix} = \begin{pmatrix} 0 \\ 13 \end{pmatrix}$$

with the restriction $c^2+s^2=1$, except in order to determine the signs, all we need is the single equation to form the 0:

$$5c = 12s,$$

So

$$25c^2 = 144 - 144c^2,$$

or

$$c = \pm 12/13,$$

and

$$s = \pm 5/13.$$

Because $5c = 12s$, the signs must be the same, and because $5s + 12c = 13$, both must be positive, so the matrix is

$$Q = \begin{bmatrix} \frac{12}{13} & -\frac{5}{13} \\ \\ \frac{5}{13} & \frac{12}{13} \end{bmatrix}.$$

You can recognize this as the rotation of the 5-12-13 right triangle onto its hypotenuse.

5.2 It is relatively easy to find three vectors that satisfy these conditions for any one distance compared with the Euclidean distance. This can easily be done with 2-vectors; for example, with $x_1 = (1,1)$, $x_2 = (1,5)$, and $x_3 = (4,4)$, the Euclidean distance is less between x_1 and x_2 than between x_2 and x_3; but the max absolute distance is greater between x_1 and x_2 than between x_2 and x_3. Similarly with $x_1 = (1,1)$, $x_2 = (4,4)$, and $x_3 = (1,6)$, the Euclidean distance is less between x_1 and x_2 than between x_2 and x_3; but the Manhattan distance is greater between x_1 and x_2 than between x_2 and x_3.

To find three vectors that satisfy the conditions simultaneously for all of the other distances compared with the Euclidean distance requires some work. I am not sure whether it's possible with 2-vectors. For 3-vectors, as the exercise requires, here are three:

$$x1 = (9, 16, 7)$$

$$x2 = (2, 2, 12)$$

$$x3 = (14, 14, 12).$$

The best way to find three is to use the computer. Here are some R fnctions to compute the distances:

```
eucliddistance<-function(x,y,d){
     d<-sqrt(sum((x-y)^2))
     return(d)
     }
maxabsolutedistance<-function(x,y,d){
     d<-max(abs(x-y))
     return(d)
     }
sumabsolutedistance<-function(x,y,d){
     d<-sum(abs(x-y))
     return(d)
     }
minkowskidistance<-function(x,y,p=1,d){
     d<-(sum((abs(x-y))^p))^1/p
     return(d)
     }
canberradistance<-function(x,y,d){
```

```
        temp<-ifelse (abs(x)+abs(y)>0,abs(x-y)/(abs(x)+abs(y)),0)
        d<-sum(temp)
        return(d)
        }
    corrdistance<-function(x,y,d){
        d<-1-abs(cor(x,y))
        return(d)
        }
    angledistance<-function(x,y,d){
        d<-1-sum(x*y)/sqrt(sum(x*x)+sum(y*y))
        return(d)
        }
    hammingdistance<-function(x,y,d){
        ix<-as.integer(x)
        iy<-as.integer(y)
        ixbits<-intToBits(ix)
        iybits<-intToBits(iy)
        ix<-as.integer(ixbits,32)
        iy<-as.integer(iybits,32)
        d<-sum(abs(ix-iy))
        return(d)
        }
    binarydistance<-function(x,y,d){
        ix<-as.integer(x)
        iy<-as.integer(y)
        ixbits<-intToBits(ix)
        iybits<-intToBits(iy)
        ix<-as.integer(ixbits,32)
        iy<-as.integer(iybits,32)
        denom<-sum(pmin(1,ix+iy))
        if (denom>0) d<-sum(abs(ix-iy))/denom else d<-0
        return(d)
        }
```

5.5(a) A metric is any function that satisfies the four properties on page 109.

Euclidean distance

1. the sum of squares is nonnegative (and the square root is the nonnegative square root).
2. if the vectors are equal (all elements are equal) the differences are all zero, and so the distance is zero; if not, it is positive.
3. the distance is symmetric, because addition is commutative.
4. the triangle inequality holds (this is easy to see just using the definition – use the squares of the distances)

max absolute difference

1. the absolute differences are nonnegative and so the maximum one is nonnegative.
2. if the vectors are equal (all elements are equal) the differences are all zero, and so the distance is zero; if not, it is positive.
3. the distance is symmetric, because addition is commutative.
4. the triangle inequality holds (this is the simple triangle inequality for absolute values).

Manhattan distance

1. the sum of absolute differences is nonnegative.
2. if the vectors are equal (all elements are equal) the differences are all zero, and so the distance is zero; if not, it is positive.
3. the distance is symmetric, because addition is commutative.
4. the triangle inequality holds (this is the simple triangle inequality for absolute values).

Minkowski distance

1. the sum of powers absolute differences is nonnegative.
2. if the vectors are equal (all elements are equal) the differences are all zero, and so the distance is zero; if not, it is positive.
3. the distance is symmetric, because addition is commutative.
4. the triangle inequality holds (this is quite difficult to show)

Canberra distance

1. the sum of absolute differences is nonnegative, and the denominator is positive (or the distance is defined to be 0).
2. if the vectors are equal (all elements are equal) the differences in the numerator are all zero, and so the distance is zero; if not, it is positive.
3. the distance is symmetric, because addition is commutative.
4. the triangle inequality holds (this is similar to the Manhattan distance)

Hamming distance

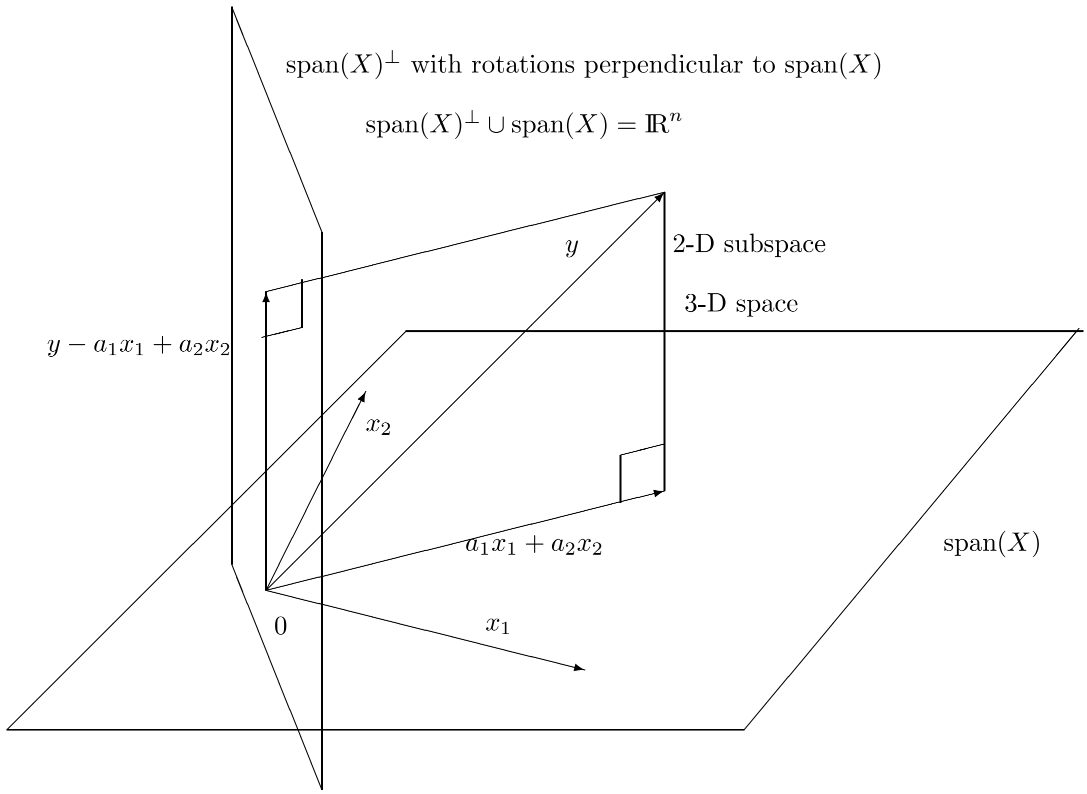
1. this is the number of differences, therefore it is nonnegative.
2. if the vectors are equal (all elements are equal) there are no differences, and so the distance is zero; if not, it is positive.
3. the distance is symmetric, because the number of differences is the same in either direction.
4. the triangle inequality holds (this is shown by counting the number of changes required to morph one vector into another).

binary distance

1. this is the proportion of differences, therefore it is nonnegative.
2. if the vectors are equal (all elements are equal) there are no differences, and so the distance is zero; if not, it is positive.
3. the distance is symmetric, because the number of differences is the same in either direction.
4. the triangle inequality holds (this is similar to the Hamming distance except that it is scaled by the number of ones).

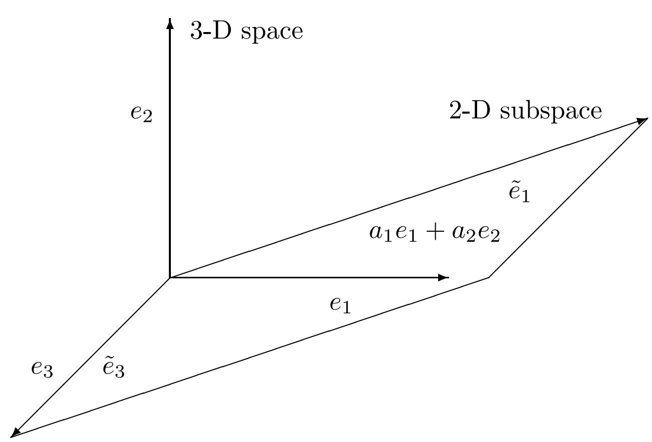
5.5(b) The only one based on norm is Minkowski, which, of course includes Euclidean, max absolute value, and Manhattan.

5.8 First, get the picture in mind in both n-space and 3-space.



$\mathbb{R}^n$

Exercise 5.8 with $X = [x_1|x_2]$ and $y = x_3$



$\mathbb{R}^3$