Astro 530: 2005 Midterm Exam - In-class: J. Wallin

Write your name and student ID number *at the top of each page of the exam.*

There are 100 points for the exam. The time for the test is $\sim$ 2.5 hours. **To receive credit on the problems, you must show your work!**

1. For this problem, assume a gas with a temperature of 7000K and an electron pressure of Pe $= 1 \times 10^3$ at $\tau = 2/3$. Sulfur has a first ionization potential of $\chi_{\rm ion} = 10.4$ eV and H has an ionization potential of $\chi_{\rm ion} = 13.6$ eV. Sulfur has an abundance of $10^{-4.8}$ times that of Hydrogen. You may assume the star is 16.3 pc away, and is visible from the southern sky about 9pm in mid-October. The statistical weight (g) for the ground and first ionized states of Hydrogen are 2 and 1. The statistical weights for Sulfur for the ground and first ionized states are 9 and 4.

   (a) (5 pts) Describe the difference between bound-bound and bound-free transitions, and what photon energies are needed to cause these processes to occur.

   Bound-bound emission occurs when an electron goes between two bound energy levels. A photon must have a specific energy to induce bound-bound transitions. Bound-free emission occurs when an electron is ionized from an atom. Any photon above a threshold energy can induce bound-free transitions.

   (b) (10 pts) What are the photon wavelengths associated with the bound-free emission in the ground states of Sulfur and Hydrogen?

$$\lambda = hc/\chi_{ion}$$

   so H can be ionized by photons with wavelength less than 911 A and S can be ionized by wavelengths less than 1191 A.

   (c) (10 pts) What fraction of the Hydrogen is in the n=2 excited state at $\tau = 2/3$?

   We use the Boltzmann equation-

   $\chi_{ion} = 1.36$

   so the excitation energy $= 13.6(1 - 1/n^2) = 10.2\ eV.$

$$\frac{N_n}{N_1} = \frac{g_n}{g_1} e^{-\chi_n/kT}$$

$$\frac{N_2}{N_1} = \frac{2(2^2)}{2} e^{-10.2\times1.6\times10^{-12}/(1.38\times10^{-16}(7000))} = 1.8^{-7}$$

   $\log N_2/N_1 = -6.7$

(d) (10 pts) When, if ever, does Sulfur play an important role in the continuous absorption coefficient in a 7000 K star?

When photons have a $911 < \lambda < 1191A$, it can ionize all atoms of Sulfur, but they cannot ionize the n=1 state of Hydrogen. Only a a very tiny fraction of H can be ionize since the atom must be in the N=2 level for this to happen. Since H is ($10^{4.8}$) 63,000 times more abundant than S, S will be about 340 times more likely to absorb compared to S.

2. Cygnus has absolute and apparent visual magnitudes of $M_v = -7.2$ and $m_v = 1.3$ with colors of $B - V = 0.1$ and a bolometric correction of $BC = 0.0$. Using reasonable assumptions, *estimate* the following quantities. Explicitly state any assumptions you are making to do these calculations and why they are reasonable.

(a) (5 pts) distance to Cygnus

using

$$m - M = 5 \log d(pc) - 5$$

$$-7.2 - 1.3 = 5 \log d(pc) - 5$$

$$13.5/5 = -\log d(pc)$$

d = 501 pc

(b) (5 pts) approximate temperature of Cygnus

Based on the B-V color, the temperatures is about 8500K.

(c) (5 pts) physical radius of Cygnus

To calculate this, we need to us

$$L = 4\pi R^2 \sigma T^4$$

So we need to find the luminosity. To find this, we can use

$$m_1 - m_2 = -2.5 \log \frac{\pi f_1}{\pi f_2}$$

since we know the luminosity of the Sun, and the absolute magnitude of the Sun and of the Cygnus, we can find the luminosity of Cygnus

$$m_{Cygnus} - m_{\odot} = -2.5 \log \frac{f_{Cygnus}}{L_{\odot}}$$

$$-7.2 - 4.82 = -2.5 \log \frac{L_{Cygnus}}{L_\odot}$$

Cygnus is 64000 times more luminous than the Sun. or $L = 2.57 \times 10^{38}$ *erg* s^{-1}.

$$2.57 \times 10^{38} = 4\pi R^2 (5.67 \times 10^{-5} (8500)^4$$

R= 8.3×10^{12} *cm = 118 solar radii*

(d) (5 pts) angular size of Cygnus

Angular size is simply

$$\alpha = 2r/d$$

where r = stellar radii and d = distance. The distance is 500 pc $= 500 \times 3 \times 10^{18} = 1.5 \times 10^{21}$ *cm*

The radius is R= 8.3×10^12 *cm.*

So $\alpha = 2R/d = 1.1 \times 10^{-8}$ *radians, or* 0.002 *arcsec.*

(e) (5 pts) how much energy per cm^2 are received on Earth from Cygnus (ignore the atmosphere of Earth)

We know that a star with a Bolometric magnitude of -26.85 (like the Sun) has a solar constant of 1.38×10^6 *erg/s. This star has a Bolometric magnitude of 1.3. Note: we could also use the fact that zero magnitude star has a flux of* 3.6×10^{-9} *erg* cm^{-2} s^{-1} *Angstrom*$^{-1}$

$$m_{Cygnus} - m_\odot = -2.5 \log \frac{\pi f_{Cygnus}}{\pi f_\odot}$$

$$1.3 - (-26.85) = -2.5 \log \frac{\pi f_{Cygnus}}{\pi f_\odot}$$

$\pi f_{Cygnus} = 5.5 \times 10^{-12} \pi f_\odot = 7.6 \times 10^{-6}$ *erg* s^{-1}

(f) (5 pts) how many photons per second per cm^2 are received at Earth in the V band from Cygnus?

We need to make an assumption here that most of the energy of the star comes out at a wavelength in the visual, say 5000 A.

This is an approximation, but the temp does peak in the visual range, so this isn't too bad.

Then we calculate the energy per photon at 5000 A.

$$E = hc/\lambda = 4 \times 10^{-12} \text{ ergs}^{-1}$$

Since we are receiving an energy of 7.6×10^{-6} *erg* s^{-1}*, we can calculate that we receive about* 1.9×10^6 *photons per second per cm squared at Earth.*

3. Radiative Transfer - *be brief but concise in any explanations you write. Feel free to sketch any diagrams that would be helpful.*

   (a) (5 pts) Write down the transfer equation for a plane parallel atmosphere.

$$\cos\theta \frac{dI_\lambda(\theta)}{d\tau_\lambda} = I_\lambda(\theta) - S_\lambda$$

   (b) (5 pts) Show that for a grey atmosphere in radiative equilibrium, the mean intensity is equal to the source function.
   First, we integrate over wavelength distribution

$$\int_0^\infty \cos\theta \frac{dI_\lambda(\theta)}{d\tau_\lambda} d\lambda = \int_0^\infty (I_\lambda(\theta) - S_\lambda) d\lambda$$

   we now integrate with respect to ω

$$\int \cos\theta \frac{dI(\theta)}{d\tau} d\omega = \int (I(\theta) - S) d\omega$$

$$\frac{d}{d\tau} \int I(\theta) \cos\theta d\omega = \int I(\theta) d\omega - \int S d\omega$$

$$\frac{d}{d\tau} \pi F = 4\pi J - 4\pi S$$

$$\frac{1}{4} \frac{dF}{d\tau} = J - S$$

   Since we have radiative equilibrium, $dF/d\tau = 0$*, so*

$$\frac{1}{4} \frac{dF}{d\tau} = J - S = 0$$

   Thus, $J = S$

(c) (5 pts) Show that the source function at $\tau = 2/3$ is equal to the surface flux of a star. Explain what conditions are needed for this to occur.

We need to assume a linear source function of the form

$$S_\lambda = a_\lambda + b_\lambda \tau$$

Using the differential source function from above, we can integrate the equation using the integrating factor $e^{-\tau_\lambda \sec\theta}$ *giving us*

$$\frac{dI_\lambda(\theta)}{d\sec\theta\tau_\lambda} e^{-\tau_\lambda \sec\theta} = I_\lambda(\theta) e^{-\tau_\lambda \sec\theta} - S_\lambda e^{-\tau_\lambda \sec\theta}$$

this simplifies to

$$\frac{d(I_\lambda(\theta) e^{-\tau_\lambda \sec\theta})}{d(\sec\theta\tau_\lambda)} = -S_\lambda e^{-\tau_\lambda \sec\theta}$$

integrating over tau from infinite optical depth to the surface of the star, we obtain this simplifies to

$$\int_0^\infty \frac{d(I_\lambda(\theta) e^{-\tau_\lambda \sec\theta})}{d(\sec\theta\tau_\lambda)} d\tau = -\int_0^\infty S_\lambda e^{-\tau_\lambda \sec\theta} d\tau$$

we can integrate the first equation and obtain

$$\left[I_\lambda(\theta) e^{-\tau_\lambda \sec\theta} \right]_0^\infty = -\int_0^\infty S_\lambda e^{-\tau_\lambda \sec\theta} d\tau$$

evaluating the limits

$$I_\lambda(0,\theta) = -\int_0^\infty S_\lambda e^{-\tau_\lambda \sec\theta} d\tau$$

We then plug our linear source equation into this to obtain the intensity

$$I_\lambda(0,\theta) = -\int_0^\infty (a_\lambda + b_\lambda \tau) e^{-\tau_\lambda \sec\theta} d\tau$$

giving the result

$$I_\lambda(0,\theta) = a_\lambda + b_\lambda \cos\theta$$

finding the surface flux is then simple, since we need multiply by $\cos\theta$ *and integrate over the angles*

$$\pi F(0) = \int_{4\pi} I(0,\theta)\cos\theta d\omega$$

we only need to integrate θ *to* $\pi/2$ *since there is no intensity going into the star at the surface*

$$\pi F(0) = \int_0^{2\pi}\int_0^{\pi/2}(a_\lambda + b_\lambda\cos\theta)\cos\theta\sin\theta d\theta d\phi$$

$$\pi F(0) = 2\pi\int_0^1 (a_\lambda + b_\lambda\mu)\mu d\mu$$

simplifying to

$$\pi F(0) = \pi\left(a_\lambda + \frac{2}{3}b_\lambda\right)$$

By inspection, we can see that $\pi F = S(\tau_\lambda = 2/3)$

(d) (5 pts) Explain how we measure how the source function of a star depends on optical depth.

We measure the dependence of the intensity on θ. *This can only be done on stars where we have good angular resolution of the surface.*

Once we have this, we can fit the coefficients in a power series of giving the result

$$I_\lambda(0,\theta) = \sum_i a_{\lambda i}(i!)\cos^i\theta$$

which gives us

$$S_\lambda = \sum_i a_{\lambda i}\tau_\lambda^i$$

(e) (5 pts) Give a short and concise reason why the outer layers of the Sun (specifically, the photosphere) must be in radiative equilibrium.

The amount of energy stored in the top 100 km of the Sun is not enough to power it for more than about 20 seconds. This calculation was done in class in in the book.

Briefly, for the density of the photosphere 10^17 *particles* cm^{-3} *and a temperature of* $6000K$ *you can calculate the energy of the gas You cannot get the* $piF = \sigma T^4$ *needed for more than about 20 seconds.*

(f) (10 pts) You have just been assigned to monitor photometric changes in the star Cygnus as part of your dissertation. You will be using a CCD with photometric filters and a small telescope to conduct these measurements. How will you calibrate the photometry of your measurement?

Briefly - you need to determine the response of the CCD and the absorption of the atmosphere using standard stars.

One standards star we have discussed is Vega. By definition, it has a magnitude in all bands of 0.0*. If we monitor this star over an evening, the zenith angle it has changes. Measuring the flux is easy, since we just count the photons in the CCD from the star and divide by the exposure time. We plot the log of the observed flux as a function of the secant of the zenith angle. The intercept on the graph where the secant of theta equals zero is the brightness the star would have outside the atmosphere.*

We then measure Cygnus with another exposure and determine its intensity and zenith angle. By using the slope from our calibration star, we can find the intensity outside the atmosphere.

The magnitude can be determined by using

$$m_1 - m_2 = -2.5 \log \frac{\pi f_1}{\pi f_2}$$

Every time we change photometric bands, we need to use a different calibration set. Atmospheric absorption and the sensitivity of the equipment is wavelength dependent.

Also, this is a very simple view of what needs to be done. We should also flat field the CCD, and remove any electronic bias it has. These are usually small corrections and outside the scope of this class.

Constant	**value**
Solar mass	2×10^{33} gm
Solar radius	7×10^{10} cm
Solar Luminosity	4×10^{33} erg s^{-1}
Angular Size	30 arc minutes
Solar Constant	1.38×10^{6} erg cm^{-2} s^{-1}
Absolute Magnitude (M_v)	4.82
Bolometric Magnitude (v_{BC})	-26.82
Bolometric Correction	0.30
Solar Radii	$7 \times 10^{1}0$