(x^2 - y) \, dx + x \, dy = dF

\[ \frac{\partial}{\partial x} (x^2 - y) = 1 \]

\[ \frac{\partial}{\partial y} \, y = 0 \]

The differential is not exact

b) \[ \int_{(1, 1)}^{(1, 2)} \, dF + \int_{(1, 2)}^{(2, 2)} \, dF = \int_{1}^{2} (x^2 - y) \, dx + \int_{1}^{2} \, x \, dy \]

\[ = x^2 - 1 + \frac{8}{3} - \frac{y}{3} - \frac{1}{3} + x = \frac{4}{3} \]

\[ \int_{(1, 1)}^{(2, 2)} \, dF = \int_{(1, 1)}^{(2, 2)} (x^2 - x + y) \, dx = \frac{x^3}{3} - \frac{x}{2} + \frac{1}{2} + y - \frac{x}{3} \]

\[ = \frac{10}{3} \]

\[ \int_{(1, 1)}^{(2, 2)} \, dF = \int_{1}^{2} \, (x^2 - x + y) \, dx = \frac{x^3}{3} \Big|_{1}^{2} = \frac{8}{3} - \frac{1}{3} \]

\[ = \frac{7}{3} \]

c) \[ dG = \frac{dF}{x^2} = \left( 1 - \frac{y}{x^2} \right) \, dx + \frac{dy}{x} \]

\[ \frac{\partial}{\partial y} \left( 1 - \frac{y}{x^2} \right) = -\frac{1}{x^2}, \quad \frac{\partial}{\partial x} \left( \frac{1}{x} \right) = -\frac{1}{x^2} \Rightarrow \text{dG \ is \ exact} \]
Integration along the same paths as before yields

\[
\int_{(1,1)}^{(1,2)} dF + \int_{(1,2)}^{(2,2)} dF = \int_1^2 \, dy + \int_1^2 \left(1 - \frac{2}{x^2}\right) \, dx
\]

\[
= y \bigg|_1^2 + \left(2 + x - \frac{2}{x}\right) \bigg|_1^2 = 2 - 1 + x - x + 1 - x = \frac{1}{x^2} - 1 - x + 1 - \frac{1}{x}
\]

\[
\int_{(1,1)}^{(2,1)} dF = \int_{1}^{2} \left(1 - \frac{1}{x^2}\right) \, dx + \int_{1}^{2} \, \frac{dy}{x} = \frac{1}{x} \bigg|_1^2 = 2 - 1 = \frac{1}{x^2} - 1 - x + 1 - \frac{1}{x}
\]

\[
\int_{(2,1)}^{(2,2)} dF = \int_{1}^{2} \left(1 - \frac{1}{x^2} + \frac{1}{x}\right) \, dx = x \bigg|_1^2 = 2 - 1 = 1
\]

All of these results for the integral agree.
We have from the first law:

$$dE = dQ - p\,dv$$

The mechanical work done by the system is given by

$$dW = p\,dv$$

From point A to point B we have

$$W_{AB} = \int_{A}^{B} F\,dv$$

which is the area under the curve.

From point B to point A we have

$$-W_{BA} = \int_{B}^{A} F\,dv$$

which is the area under the lower curve.

The net work done by the system is therefore

$$W = W_{AB} + W_{BA} = \text{area within figure between the two curves.}$$